Título del trabajo: Toward determining the behavior of fragmentation functions during the impact crushing of minerals

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ABSTRACT

The subject of this paper is particle population balance models (PBM) that describe mineral impact crushing behavior; it is mainly concerned with the selection and fragmentation distribution function due to their importance in this context. The data obtained by Datta (1999) by applying impact pendulum testing to single limestone particles, used by Austin (2002, 2004a and 2004b) to obtain the principal parity of fragmentation functions, were processed in order to compare them with the results of other authors, such as Nikolov (2002 and 2004) and Vogel and Peukert (2003 and 2005) so as to increase knowledge of impact crushing. From this data it has been possible to obtain the performance models of the principal parameters of fragmentation function, thereby reducing the number of experiments necessary to model the impact crushing process. The authors also propose a simplification of the selection function model posited by Austin (2002).

Key words: Impact crushing, Fragmentation, Modeling and simulation.

Introduction

The methods of particle population balance (PBM) represent very useful tools for the modeling and simulation of the processes of mineral crushing and grinding. These methods are based on the fragmentation distribution function and the selection or classification function, both initially formulated by Epstein in 1948. The fragmentation distribution function describes the redistribution of the particles obtained by the fragmentation of larger particles and the selection or classification function quantifies the probability of particles of each size being fragmented in determined processing conditions.

To carry out the definition of these functions, different methodologies have been established. Some of them are based on single-particle tests or mill tests at laboratory scale (Gardner and Austin, 1962; Kelsall, 1965; Herbst and Fuerstenau,1968; Austin and Luckie, 1971 and 1972; Gardner and Sukanjnajtee, 1972; Kapur, 1982). Other methodologies are proposed to work at industrial scale (Austin and Klimpel, 1964; Kapur and Agrawal, 1970; Malghan and Fuerstenau, 1976; Austin et al., 1982a and 1982b; Austin and Brame, 1983).

In the case of impact crushing, the main models published at industrial scale are those of Nikolov (2002, 2004) and Vogel & Peukert (2004). The main work carried out using single-particle tests was developed by Austin (2002; 2004a and 2004b). This approach is considered of huge importance in determining the principal elements of the size reduction, being its main disadvantage is that these procedures are relatively labor intensive and any reduction in the number of experiments would be of considerable practical interest.

Materials and methods

Experimental data obtained by Datta (1999) with a single particle impact test device for limestone samples were used (see Table 1). These data were those used by Austin (2002, 2004a and 2004b) to determine the main elements of the impact crushing function. Austin's data and results were processed to compare the behavior of the fragmentation functions during impact crushing, as reported by this author, with the results reported by Nikolov (2002, 2004) and Vogel and Peukert (2003, 2005). The behavior of the parameters of the fragmentation functions (Austin, 2004b) is laid out in Table 2.

In order to study the behavior of the principal parameters of the fragmentation function and of the simplified classification function, it was necessary to take into account the principal tendencies of impact crushing, as reported by the experimental data of different authors.

In order to obtain the parameters of these models, adjustment procedures were carried out using commercial Matlab 7.0. All of the proposed models were also checked with various statistical criteria using StatGraphics 5.1.

Tabla 1. Comportamiento de las energías máxima y mínima para diferentes tamaños de partículas obtenidas vía el test de impacto (Datta, 1999)

Table 1 — Behavior of maximum and minimum energy for different sizes of particles obtained via impact test (Datta, 1999).

Size (mm)	E_{max}(J/g)	E_{min}(J/g)
19.2	1.61	0.023
13.6	1.97	0.028
9.6	2.41	0.034
6.8	2.95	0.042
4.8	3.6	0.051
3.4	4.41	0.063
2.4	5.39	0.076
1.7	6.59	0.093
1.2	8.05	0.11
0.85	9.85	0.14
0.6	12	0.17
0.424	14.7	0.21
0.3	18	0.26
0.212	22	0.31
0.15	26.9	0.38
0.106	32.9	0.47
0.075	40.2	0.57
0.053	49.2	0.7
0.037	60.1	0.85
0.027	73.5	1.04
0.019	89.9	1.28
0.013	110	1.56
0.009	134	1.91

Tabla 2. Parámetros de fragmentación

Table 2 — Parameters of fragmentation functions.

Specific energy of impact (J/kg)	α	β	ϕ
Fragmentation distribution function			
180	0.96	16	0.18
300	0.96	7	0.2
800	0.96	3.4	0.3
1,500	0.96	2.4	0.42
2,000	0.96	2	0.52
2,730	0.96	2	0.67
Selection function			
A = 1/0.235	C = 130 J/kg		
m = 0.58	x_0 = 1 mm		

ANALYSIS OF RESULTS

Fragmentation distribution function. There are various expressions available to represent the fragmentation distribution function, but that proposed by Austin and Luckie (1972) is usually considered the more adequate in the case of impact crushing (Austin, 2002; 2004a and 2004b; Nikolov, 2004). This expression in shown in Eq. (1).

$$B(x,y) = \phi \cdot \left(\frac{x}{y}\right)^{\alpha} + (1-\phi) \cdot \left(\frac{x}{y}\right)^{\beta} \quad (1)$$

where x is the size of the particle obtained from the fragmentation of particles with size y, α is a constant, β and Φ may be considered as constants (Nikolov, 2004) or dependent on the specific energy of fragmentation (Austin, 2002; 2004a and 2004b).

Figure 1 shows Eq. (1) (following Prasher, 1987), in which αis constant, β represents the formation of the coarsest particles and Φ describes the formation of finer particles.

Narayanan and Whiten (1988) reported that the influence of Φ in the fragmentation distribution function increases as the energy of impact also increases. This is coincident with Nikolov (2002) and Austin (2002; 2004a and 2004b). Taking this into account, alongside the interpretation of the parameters β and Φ and their values as shown in Table 2, two expressions can be proposed for the definition of these parameters of the fragmentation distribution function from the specific energy of impact E, which are shown in Eqs. (2a) and (2b). The models proposed yield the parameters from the reference values E_0, β_0 (Nikolov, 2004)

$$\phi = k_1 \cdot \left(\frac{E}{E_0}\right) + C_1 \qquad (2a)$$

$$E_0 < E;$$

$$\beta = \frac{k_2}{\beta_0} \cdot \left[\ln(E) - C_2\right] \qquad (2b)$$

where E is the specific energy of impact; E0 and β0 the reference values and k_1, k_2, C_1 and C_2 are constants.

Table 3 and Fig. 2 show the values obtained by Austin (see Table 2) with a continuous line and the values obtained using Eqs. (2a) and (2b) with symbols. It can be observed that the representation yields a straight line with a good correlation factor, so the determination of the parameters can be made with a minimum of tests with enough reliability.

Selection function. The model proposed by Austin to describe fragmentation is represented in Eqs. (3) and (4).

$$C(x_i) = \bar{A} \cdot \ln\left(\frac{E_k}{K_i}\right) \qquad (3)$$

where $\bar{A}$ is a dimensionless constant of the material, E k is the specific energy and K i is obtained with Eq. (4):

$$K_i = C \cdot \left(\frac{x_i}{x_0}\right)^{-m} \qquad (4)$$

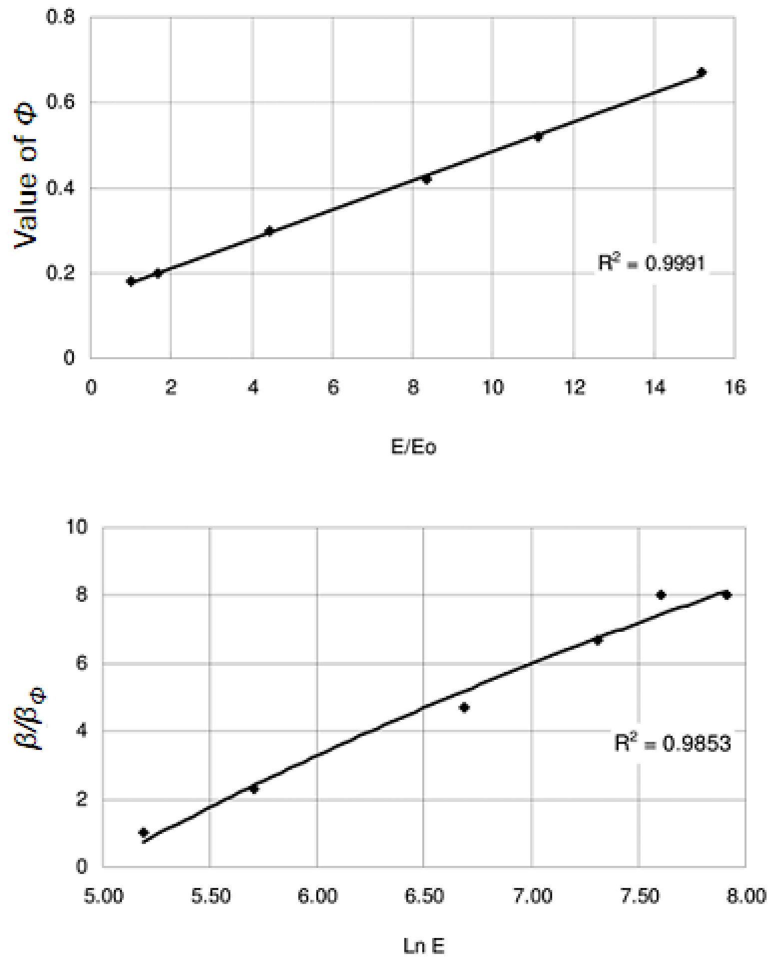

Figure 2 — Comparison of parameters β and ϕ from Table 2 and those obtained via simulation.

C and m are constants that are dependent on the material and x_0 is the size of the reference particles.

By using Eqs. (3) and (4) with the parameter values , C, m and x_0 , as determined by Austin (see Table 2), for some of the energy values used by this author, it is possible to obtain the probability of fragmentation for different sizes of particles.The results of these calculations are shown in Table 4.

Fig. 3 represents the data of Table 4. From the curves represented and the equations of the fittings shown in the graph, the steepness of the linear part of the curve is the same in all

cases, at different values of specific impact energy. This result indicates that this parameter seems to be independent of the specific energy of impact and this obtained result agrees with the predictions made by Nikolov (2002, 2004).

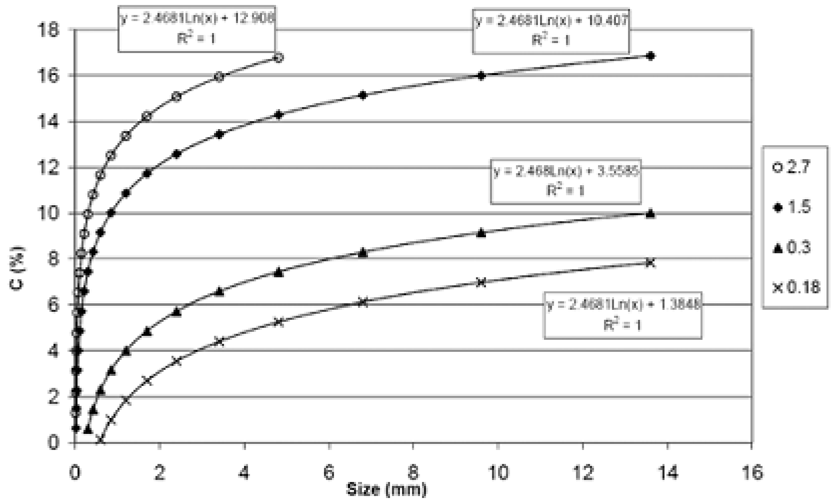

Figure 3 — Representation of the selection function following Austin's model for four different energy-of-impact values.

To compare the selection function models obtained by Nikolov (2002, 2004), Vogel and Peukert (2003, 2005) and Austin (2002), the values of Table 4 were fitted to the models of Nikolov (Eq. 5) and Vogel & Peukert (Eq. 6).

$$C(x_i) = 1 - \exp\left\{-\left[\frac{xi - d}{d}\right]^k\right\} \qquad (5)$$

$$C(x_i) = 1 - \exp\left[-f_{mat} \cdot x_i \cdot \left(E - E_{\min}\right)\right] \qquad (6)$$

Although the results do not it Eq. (5) as well as expected, they show a similar trend. The discrepancy would seem to be caused by the difference between the particle population balance models used by both authors.

In the case of Eq. (6), the results showed quantitative differences, probably due to the assumption that Emin x = constant and fmat= constant.

Table 5 shows the behavior of size and minimum impact energy as generated from the data used by Datta (1999), on which the model shown in Fig. 4 is based. Considering that the equipment used in Datta's test and those used by Vogel and Peukert (2003) are similar and based in both cases on single-particle tests, the cause of the limitations of Vogel and Peukert's model must stem from the size of the data sample since, in specific ranges, they show similar results. By approximating Austin's results with the model of Eq. (6) and considering f_{mat} as not a constant, both models show a high degree of coincidence in their results.

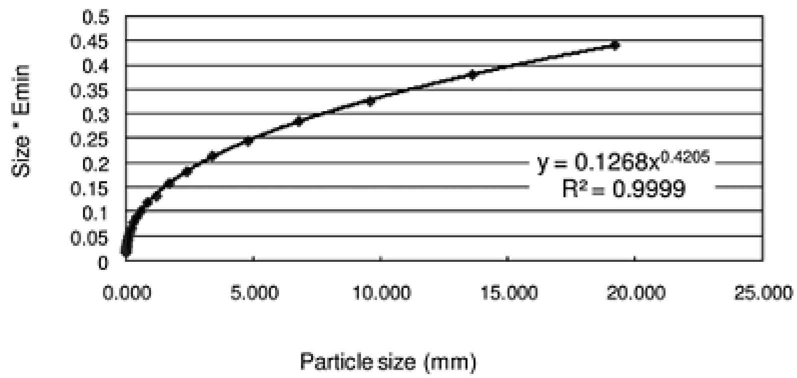

Figure 4 — Behavior of size of particle product in relation to the minimum energy of fragmentation according to Datta's (1999) data.

All the analysis carried out up to date validates the Austin model for determining the selection function during impact crushing. Nevertheless, the number of parameters in this model can be reduced if represented as shown in Eq. (7),

$$C(x_i) = B \cdot \ln\left(\frac{x_i}{d}\right)^n \qquad (7)$$

where B and n are constant values.

Tables 5 and 6 show the comparison of results obtained by means of Eq. (7) and those obtained by means of Eqs. (3) and (4). It can be observed that in no case do the differences

between each model reach 1%.

From the analysis of Nikolov results (2004), it becomes possible to affirm that, for other conditions of constant operations, the particle size for which the fragmentation value is zero can be determined from the reference parameters by means of Eq. (8):

Table 5 — Behavior of the size of particle product in relation to the minimum energy of fragmentation according to Datta's (1999) data.

x	19.20	13.60	9.60	6.80	4.80	3.40	2.40
E_{min}	0.023	0.028	0.034	0.042	0.051	0.063	0.076
$x \cdot E_{min}$	0.441	0.380	0.326	0.285	0.244	0.214	0.182
x	1.70	1.20	0.85	0.60	0.42	0.30	0.212
E_{min}	0.09	0.110	0.14	0.170	0.210	0.26	0.310
$x \cdot E_{min}$	0.158	0.132	0.119	0.102	0.089	0.078	0.065

$$d = d_1 \cdot \left(\frac{E_0}{E_i} \right)^l \qquad (8)$$

Equations (8) and (9) represent the principal relative behavior trends which should correspond to the particle sizes whose probability of fragmentation is minimal or maximal, depending on the energy of impact. A comparison of the values obtained by means of simulation and the experimental values of Table 1 are shown in Tables 7 and 8 and in Figs. 6 and 7. Thus, it can be seen that there is a high degree of correlation between the experimental and the calculated results.

Conclusions

The parameters of the model of the fragmentation distribution function can be obtained with a minimum of experimentation by means of the approximations presented.

From the analysis carried out of the different formulations of the selection function, it can be concluded that these formulations depend on the method used for their determination, something which must be taken into consideration during the modeling and simulation of impact fragmentation processes.

The simplified model proposed can yield an approximation as good as that from the Austin model, but with a reduced number of parameters.

Table 7 — Comparison of experimental and calculated values of *D*.

D (mm)	*E* (J/g)	E_1/E	K_1	D_{calc}	Difference
19.2	1.61	1.000	1.72018	19.2	0.000
13.6	1.97	0.817	1.72018	13.57	0.031
9.6	2.41	0.668	1.72018	9.59	0.007
6.8	2.95	0.546	1.72018	6.77	0.025
4.8	3.60	0.447	1.72018	4.81	0.010
3.4	4.41	0.365	1.72018	3.39	0.007
2.4	5.39	0.299	1.72018	2.40	0.002
1.7	6.59	0.244	1.72018	1.70	0.000
1.2	8.05	0.200	1.72018	1.20	0.005
0.85	9.85	0.163	1.72018	0.85	0.002
0.6	12.0	0.134	1.72018	0.61	0.006
0.424	14.7	0.110	1.72018	0.43	0.004
0.3	18.0	0.089	1.72018	0.30	0.002
0.212	22.0	0.073	1.72018	0.21	0.002
0.15	26.9	0.060	1.72018	0.15	0.001
0.106	32.9	0.049	1.72018	0.11	0.001
0.075	40.2	0.040	1.72018	0.08	0.001
0.053	49.2	0.033	1.72018	0.05	0.001
0.037	60.1	0.027	1.72018	0.04	0.001
0.027	73.5	0.022	1.72018	0.03	0.000
0.019	89.9	0.018	1.72018	0.02	0.000
0.013	110	0.015	1.72018	0.01	0.000
0.009	134	0.012	1.72018	0.01	0.001

Table 8 — Comparison of experimental and calculated values of *D*.

D (mm)	*E* (J/g)	E_2/E	K_2	d_{calc}	Difference
19.2	0.023	1.000	1.7352305	19.20	0.000
13.6	0.028	0.821	1.7352305	13.65	0.048
9.6	0.034	0.676	1.7352305	9.74	0.144
6.8	0.042	0.548	1.7352305	6.75	0.047
4.8	0.051	0.451	1.7352305	4.82	0.022
3.4	0.063	0.365	1.7352305	3.34	0.058
2.4	0.076	0.303	1.7352305	2.41	0.013
1.7	0.093	0.247	1.7352305	1.70	0.000
1.2	0.11	0.209	1.7352305	1.27	0.070
0.85	0.14	0.164	1.7352305	0.84	0.014
0.6	0.17	0.135	1.7352305	0.60	0.003
0.424	0.21	0.110	1.7352305	0.41	0.010
0.3	0.26	0.088	1.7352305	0.29	0.014
0.212	0.31	0.074	1.7352305	0.21	0.002
0.15	0.38	0.061	1.7352305	0.15	0.002
0.106	0.47	0.049	1.7352305	0.10	0.004
0.075	0.57	0.040	1.7352305	0.07	0.002
0.053	0.70	0.033	1.7352305	0.05	0.002
0.037	0.85	0.027	1.7352305	0.04	0.000
0.027	1.04	0.022	1.7352305	0.03	0.001
0.019	1.28	0.018	1.7352305	0.02	0.001
0.013	1.56	0.015	1.7352305	0.01	0.000
0.009	1.91	0.012	1.7352305	0.01	0.000
Summary error					0.458

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